Reg No.:_______________ Name:__________________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
FOURTH SEMESTER B.TECH DEGREE EXAMINATION(R&S), MAY 2019

Course Code: MA204
Course Name: PROBABILITY, RANDOM PROCESSES AND NUMERICAL METHODS (AE, EC)

Max. Marks: 100 Duration: 3 Hours

Normal distribution table is allowed in the examination hall.

PART A

Answer any two full questions, each carries 15 marks

1 a) The probability mass function of a random variable X is given below: (7)

x	0	1	2	3
$f(x)$	c	$2c^2$	c^2	$3c^2$

Determine (i) the value of c (ii) $P(X \geq 1)$

(iii) $P[X > 1/(0 < X < 3)]$ (iv) $E(X)$

b) The probability of an item produced by a certain machine will be defective is 0.05. If the produced items are sent to the market in packets of 20, find the number of packets containing (i) atleast 2 (ii) exactly 2 (ii) atmost 2 defective items in a consignment of 1000 packets using Poisson distribution. (8)

2 a) The mileage which a car owner gets with a certain kind of tyre is a random variable having an exponential distribution with mean 60,000 km Find the probabilities that one of these tyres will last, (7)
(a) at least 55,000 km (b) atmost 65,000 km

b) A random variable X $follows$ uniform distribution in $(-3,3)$. Find (8)

(i) $P(|X| < 2)$ (ii) $P(|X - 2| < 2)$ (iii) $P(|X| > 1)$

(iv) the value of K for which $P(X > k) = \frac{1}{3}$

3 a) Fit a binomial distribution to the following data: (7)

x	0	1	2	3	4	5	6
f_i	5	18	28	12	7	6	4

b) In an examination 30% of the candidates obtained marks below 40 and 10% of the candidates got above 75 marks. Assuming that the marks are normally distributed, find the mean and standard deviation of the distribution. (8)

PART B

Answer any two full questions, each carries 15 marks

4 a) The life time of a certain type of electric bulbs may be considered to follow exponential distribution with mean 50 hrs. Use central limit theorem to find the approximate probability that 100 of these electric bulbs will provide a total of more than 6000 hrs of burning time. (7)

b) The joint density function of two continuous random variables X, Y is given by (8)

$$f(x,y) = \begin{cases} K(1-x-y), & 0 < x < \frac{1}{2}; 0 < y < \frac{1}{2} \\ 0, & otherwise \end{cases} .$$

Find (i) the value of K $(ii) P\left(X < \frac{1}{4}, Y > \frac{1}{4}\right)$ (iii) the marginal distributions of X, Y (iv) check whether X, Y are independent.

5 a) Let $X(t) = Acos\omega t - Bsin\omega t$, where A and B are independent random variables following $N(0, \sigma^2)$. Then show that $\{X(t)\}$ is WSS. (7)

b) Find the power spectral density function of the WSS process whose autocorrelation function is $e^{-\alpha\tau^2}$. (8)

6 a) The joint probability distribution of X and Y is given by $f(x,y) = \frac{2x+3y}{54}$ for $x = 1, 2;\ y = 1, 2, 3.$ Find (i) the marginal distributions of X and Y (ii) The conditional distribution of X for $Y = y$. (7)

b) The power spectral density of a WSS process is $\frac{\omega^2+9}{\omega^4+5\omega^2+4}$. Find the autocorrelation function and power of the process. (8)

PART C

Answer any two full questions, each carries 20 marks

7 a) The tpm of a Markov chain with 4 states 0, 1, 2, 3 is given by (10)

$$P = \begin{bmatrix} 0.2 & 0.8 & 0 & 0 \\ 0 & 0.2 & 0.8 & 0 \\ 0 & 0 & 0.2 & 0.8 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

with initial distribution $P\{X_0 = i\} = \frac{1}{3}, i = 0,1,2,3.$

Find $(i) P\{X_1 = 2/ X_0 = 1\}$ $(ii)\ P\{X_2 = 3/X_0 = 1\}$

$(iii)\ P(X_2 = 3, X_1 = 2, X_0 = 2)$ $(iv)\ P\{X_2 = 3\}$

b) The tpm of a Markov Chain is $P = \begin{bmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$. Find the steady state distribution of the chain. (5)

c) A radioactive source emits particles at the rate of 6 per minutes in accordance with Poisson process. Each particle emitted has a probability of $\frac{1}{3}$ being recorded. Find the probability that atleast 5 particles are recorded in 5 minutes. (5)

8 a) Evaluate $\int_{4}^{5.2} log_e(x)\, dx$ using Simpson's 1/3rd rule . (Take $h = 0.2$) (7)

b) Use Newton's forward interpolation formula to evaluate $y(23)$ from the following data: (7)

x	20	25	30	35	40	45
y	34.3	32.1	29.3	25.6	22.7	21.9

c) Use Runge-Kutta method of order 4 to find $y(0.2)$ for the differential equation: (6)

$y' = 3x + 0.5y,\ y(0) = 1$. (Take h = 0.2)

9 a) A house wife buys 3 types of cereals A, B, and C. She never buys the same cereals in successive weeks. If she buys cereal A, next week she buys B. However, if she buys B or C, next week she is 3 times as likely to buy A as the other cereals. In the first week of May she buys cereal C. Then what is the probability that (i) in the second week she buys cereal A (ii) In the third week she buys cereal C (iii) In the long run, how often she buys cereal B. (10)

b) Use Lagrange's interpolation formula to find $y(2)$ from the following table: (5)

x	1	3	4
y	1	27	64

c) Evaluate cube root of 41 correct to four decimal places using Newton- Raphson method correct to 4 decimal places. (5)

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