

Reg No.: _____

Name: _____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

THIRD SEMESTER B.TECH DEGREE EXAMINATION(S), MAY 2019

Course Code: MA201**Course Name: LINEAR ALGEBRA AND COMPLEX ANALYSIS**

Max. Marks: 100

Duration: 3 Hours

PART A*Answer any two full questions, each carries 15 marks*

Marks

- 1 a) Prove that the function $\sin z$ is analytic and find its derivative. (7)
- b) Under the transformation $w = \frac{1}{z}$, find the image of $|z - 2i| = 2$ (8)
- 2 a) Find the analytic function whose imaginary part is (7)
- $$v(x, y) = \log(x^2 + y^2) + x - 2y.$$
- b) Under the transformation $w = z^2$, find the image of the triangular region (8)
- bounded by $x = 1$, $y = 1$ and $x + y = 1$.
- 3 a) Show that $f(z) = \begin{cases} \frac{z \operatorname{Re}(z)}{|z|}, & z \neq 0 \\ 0, & z = 0 \end{cases}$ is not differentiable at $z = 0$ (7)
- b) Find the bilinear transformation that maps the points $-1, i, -1$ onto $i, 0, -i$. (8)

PART B*Answer any two full questions, each carries 15 marks*

- 4 a) Using Cauchy's integral formula, evaluate $\int_C \frac{e^z}{(z^2+4)(z-1)^2} dz$, where C is the (7)
- circle $|z - 1| = 2$.
- b) Evaluate $\int_0^{2+i} (\bar{z})^2 dz$ along (8)
- (i) the real axis to 2 and then vertically to $2 + i$.
- ii) the line $2y = x$
- 5 a) Find all singular points and residues of the functions (7)

(a) $f(z) = \frac{z - \sin z}{z^2}$ (b) $f(z) = \tan z$

b) Evaluate $\int_0^{2\pi} \frac{1}{5 - 3\sin\theta} d\theta$. (8)

6 a) Evaluate $\int_C \log z dz$ where C is the circle $|z| = 1$ (7)

b) Evaluate $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)(x^2+4)} dx$ (8)

PART C

Answer any two full questions, each carries 20 marks

7 a) Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 5 \\ 1 & 5 & 5 & 7 \\ 8 & 1 & 14 & 17 \end{bmatrix}$ (8)

b) Find the values of a and b for which the system of linear equations (7)

$$x + 2y + 3z = 6, x + 3y + 5z = 9, 2x + 5y + az = b$$
 has (i) no solution

(ii) a unique solution (iii) infinitely many solutions

c) Show that the vectors $[3 \ 4 \ 0 \ 1]$, $[2 \ -1 \ 3 \ 5]$ and $[1 \ 6 \ -8 \ -2]$ are linearly independent in \mathbb{R}^4 . (5)

8 a) Solve the system of equations by Gauss Elimination Method: (8)

$$3x + 3y + 2z = 1, \quad x + 2y = 4, \quad 10y + 3z = -2, \quad 2x - 3y - z = 5$$

b) Find the nature, index, rank and signature of the quadratic form (6)

$$x_1^2 + 2x_2^2 + 3x_3^2 + 2x_1x_2 - 2x_1x_3 + 2x_2x_3$$

c) Find the Eigen values and Eigen vectors of $\begin{bmatrix} 4 & 2 & -2 \\ 2 & 5 & 0 \\ -2 & 0 & 3 \end{bmatrix}$ (6)

9 a) Diagonalize the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ (8)

b) Define symmetric and skew symmetric matrices. Show that any real square matrix can be written as the sum of a symmetric and a skew symmetric matrix. (6)

c) What type of conic section is represented by the quadratic form (6)

$$3x^2 + 22xy + 3y^2 = 0$$
 by reducing it into canonical form.
